Development of a MAV---
Modeling, Control and Guidance

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ENSICA

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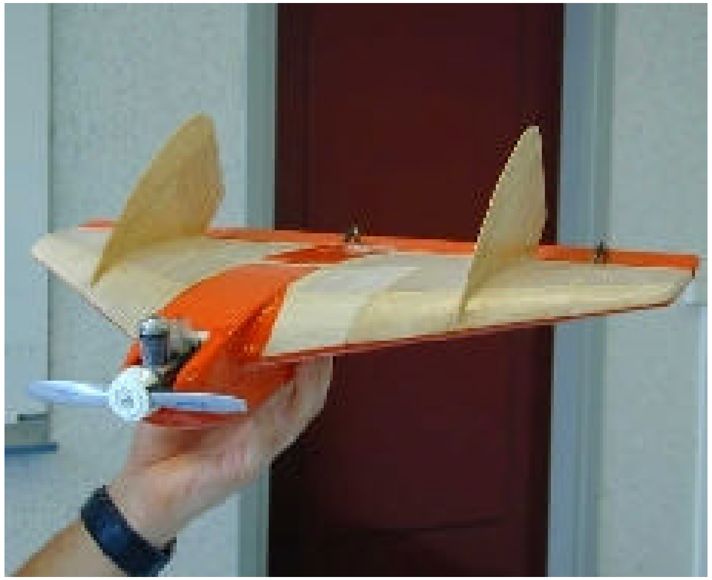
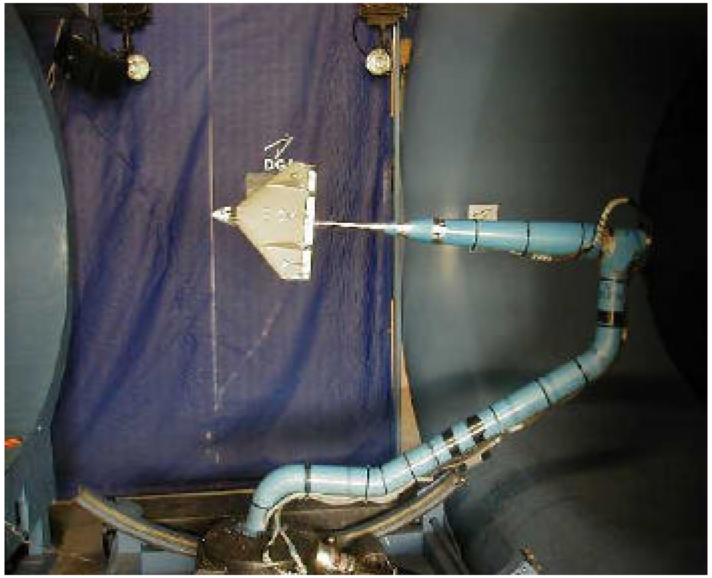
- 3 Modeling of the MAV
- 4 Control Law Design
- 5 Guidance Law Design
- 6 Attitude Determination
- 7 Conclusion

1 Introduction

- Pictures of Pégase - 50





- The general characteristics of Pégase - 50 :

  - Wing span $b = 0.5m$
  - Length $L = 0.34m$
  - Wing area $S_{ref} = 0.0925m^2$
  - Aerodynamic mean chord $c = 0.185m$
  - Speed of cruising $V_0 = 50/60km/h$

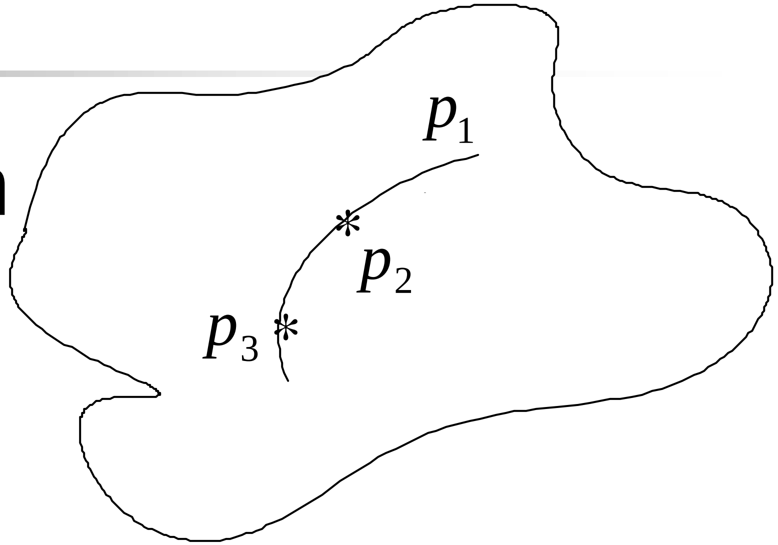
2 Wind Tunnel Test

- A wind tunnel test has been carried out in CEAT windtunnel S4. The variables of this test are:
  - Angle of attack (from -30 to 30 degrees)
  - Side-slip angle (from 0 to 45 degrees)
  - Elevator (-5, 0, 5)
  - Aileron (0, 5, 15)
  - Or aileron acts as elevator (-10, -5, 0, 5)

3 Modelling of the MAV

To fulfil the mathematic model,the force coefficients and moment coefficients are needed. They can be obtained by ***interpolation*** of different state of the wind tunnel test.

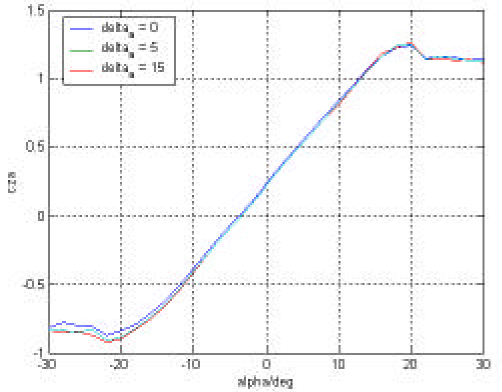
- Method of interpolation



  - ***<u>linear interpolation</u>***
  - cubic interpolation
  - nearest neighbor interpolation
  - spline interpolation

- Interpolation results of six coefficients
  - Lift coefficient

    As seen from later result (figure 1), the angle of attack, side-slip angle and elevator determine lift coefficient while the effect of aileron can be ignored. Figure 1 shows the variation of lift coefficient when the position of aileron is $0^{\circ}, 5^{\circ}, 15^{\circ}$ (the position of elevator is 0°, side-slip angle is 0°).



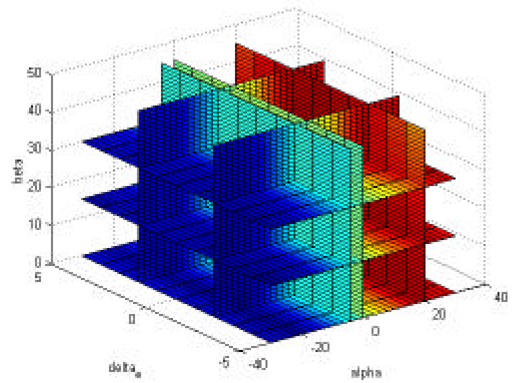


Figure 1 Relation between lift coefficient and Aileron

Figure 2 Volumetric slice plot of lift coefficient

3 Modelling of the MAV (4)

- Drag coefficient

It can be seen from figure 3, figure 4, the angle of attack, side-slip angle and elevator determine drag coefficient while the effect of aileron can be ignored. Figure 3 shows the variation of drag coefficient when the position of aileron is $0^\circ, 5^\circ, 15^\circ$ (the position of elevator is set as 0°, side-slip angle is 0°).

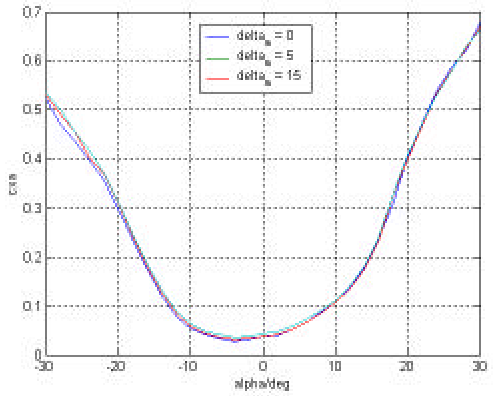


Figure 3 Relation between drag coefficient and aileron

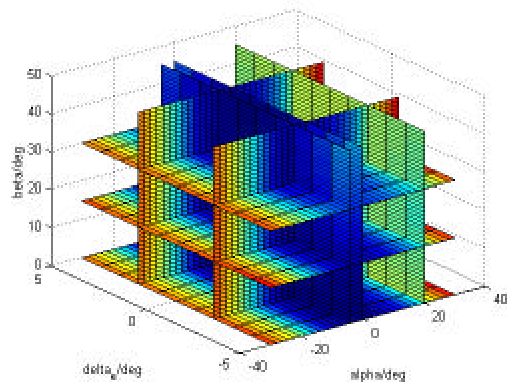


Figure 4 Volumetric slice plot of drag coefficient

3 Modelling of the MAV (6)

- lateral (side-force) coefficient

The variaton of control surfaces is of little effect on the changing of lateral coefficient, therefor two dimensional interpolation needed to obtain the coefficient.

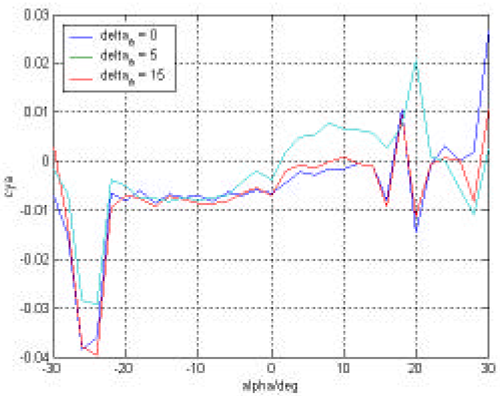


Figure 5 Relation between lateral coefficient and aileron

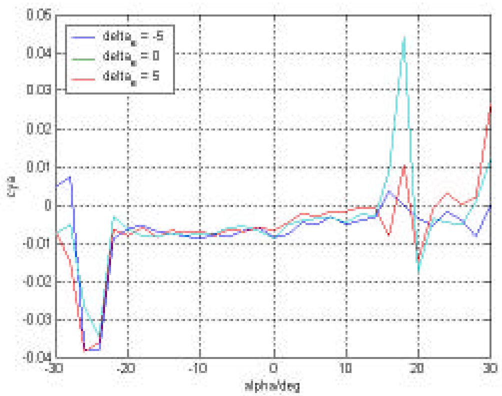


Figure 6 Relation between lateral coefficient and elevator

3 Modelling of the MAV (8)

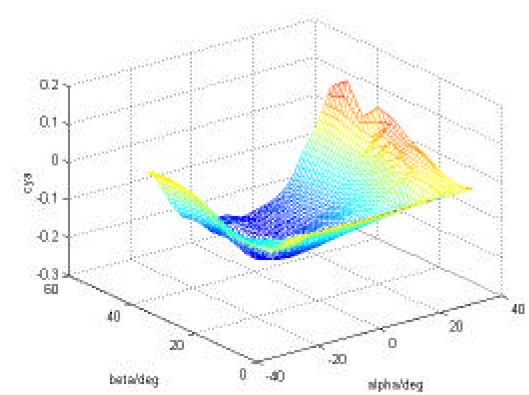


Figure 7 lateral coefficient variation

3 Modelling of the MAV (9)

- rolling moment coefficient

As the deflection of elevator almost makes no contribution to the variation of rolling moment coefficient, three dimensional interpolation is needed to acquire this coefficient.

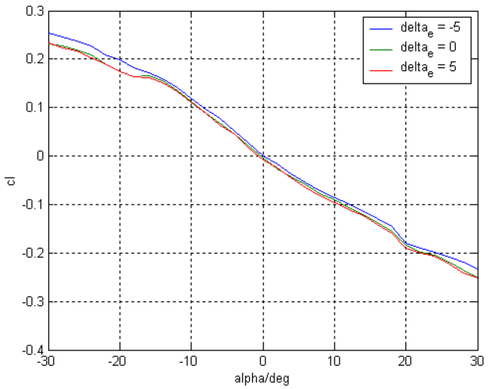


Figure 8 Relation between rolling moment coefficient and elevator

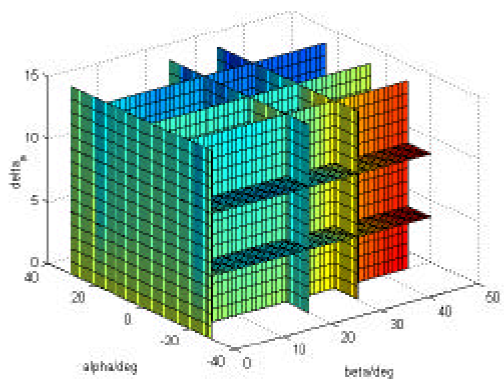


Figure 9 Volumetric slice plot rolling moment coefficient

3 Modelling of the MAV (11)

- pitching moment coefficient

Four dimensional interpolation needed to obtain pitching moment coefficient because all the control surfaces, angle of attack and side-slip angle affect it noticeably.

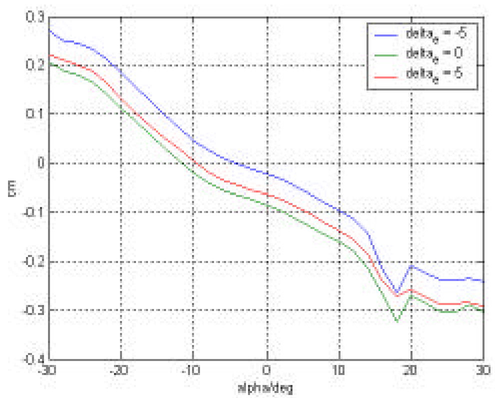


Figure 10 Relation between pitching moment coefficient And elevator

$(b = 0, d_a = 0)$

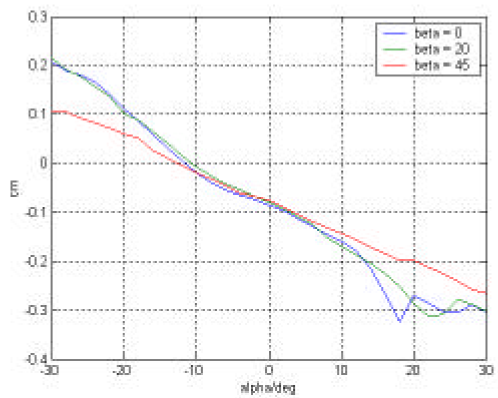


Figure 11 Relation between pitching moment coefficient and side-slip angle

$(d_a = d_e = 0)$

3 Modelling of the MAV (13)

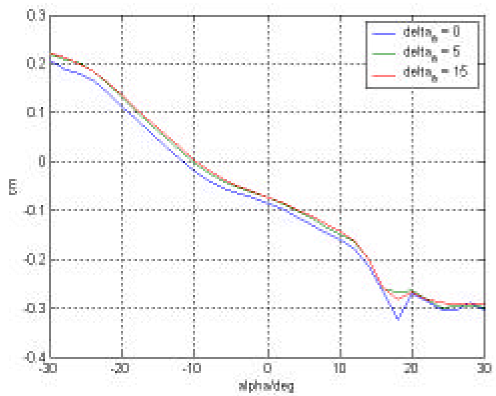
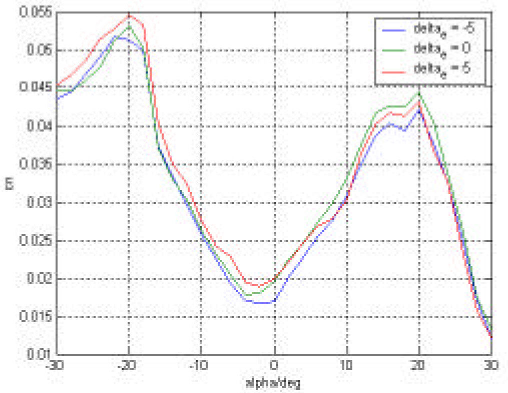


Figure 12 Relation between pitching moment coefficient And aileron $(d_e = 0, b = 0)$

3 Modelling of the MAV (14)

- yawing moment coefficient

Yawing moment coefficient is a three dimensional interpolation function of side-slip angle, angle of attack and aileron, and the effect of elevator can be ignored.



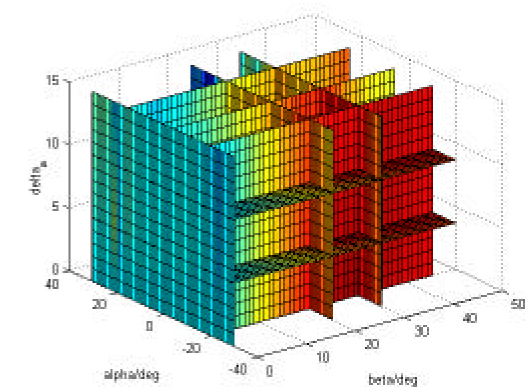
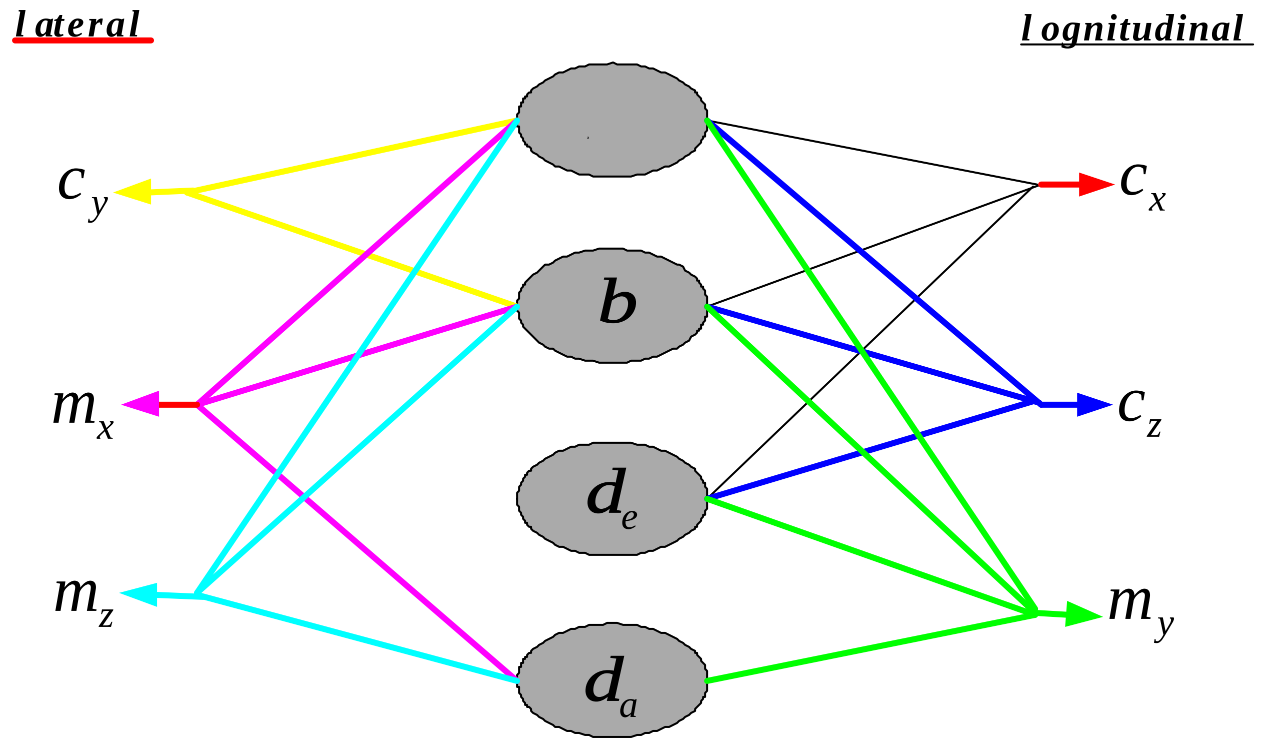


Figure 13 Relation between yawing moment coefficient and elevator $(\beta = 45^\circ, \delta_a = 0)$

Figure 14 Volumetric slice plot of yawing moment

3 Modelling of the MAV (16)



3 Modelling of the MAV (17)

- Equation of the MAV

From the coefficient obtained above and the evaluation of inertia moment, a ***twelve degrees nonlinear*** equation can be derived. The states of the equation are: three dimensional position and velocity, angular rate of body axis and three attitude angles. In this model, the disturbances of three axis wind and wind rate are included.

$$
\begin{cases}
\dot{V}_x = \dfrac{P - F_x \cos\boldsymbol{a}\cos\boldsymbol{b} - F_z \sin\boldsymbol{a} - F_y \cos\boldsymbol{a}\sin\boldsymbol{b}}{m} - g\sin\boldsymbol{q} - V_y r - V_z q \\
\dot{V}_y = \dfrac{-F_x \sin\boldsymbol{b} + F_y \cos\boldsymbol{b}}{m} + g\cos\boldsymbol{q}\sin\boldsymbol{j} + V_z p - V_x r \\
\dot{V}_z = \dfrac{-F_x \sin\boldsymbol{a}\cos\boldsymbol{b} + F_z \cos\boldsymbol{a} - F_y \sin\boldsymbol{a}\sin\boldsymbol{b}}{m} + g\cos\boldsymbol{q}\cos\boldsymbol{j} + V_x q - V_y p \\
\dot{p} = \dfrac{M_x + \Delta w_x + (I_y - I_z + \dfrac{I_{xz}^{\;2}}{I_z})qr - (I_{xz} + \dfrac{I_{xz}}{I_z(I_x - I_y)})pq + \dfrac{I_{xz}}{I_z}M_z}{I_x - I_{xz}^{\;2}/I_z} \\
\dot{q} = (M_y + \Delta w_y + (I_z - I_x)pr - I_{xz}(r^2 - p^2)/I_y \\
\dot{r} = -(-M_z + \Delta w_z - (I_x - I_y)pq + I_{xz}(-qr + \dot{p}))/I_z \\
\dot{\boldsymbol{y}} = (\boldsymbol{w}_y \cos\boldsymbol{j} - q\sin\boldsymbol{j})/\cos\boldsymbol{q} \\
\dot{\boldsymbol{q}} = -r\sin\boldsymbol{j} + p\cos\boldsymbol{j} \\
\boldsymbol{j} = p + \tan\boldsymbol{q}(+r\cos\boldsymbol{j} + p\sin\boldsymbol{j}) \\
\dot{h} = V_x \sin\boldsymbol{q} - V_z \cos\boldsymbol{q}\cos\boldsymbol{j} - V_y \cos\boldsymbol{q}\sin\boldsymbol{j} \\
\dot{x} = V_x \cos\boldsymbol{y}\cos\boldsymbol{q} - V_z(\sin\boldsymbol{y}\sin\boldsymbol{j} - \cos\boldsymbol{y}\sin\boldsymbol{q}\cos\boldsymbol{j}) + V_y(\sin\boldsymbol{y}\cos\boldsymbol{j} + \cos\boldsymbol{y}\sin\boldsymbol{q}\sin\boldsymbol{j}) \\
\dot{y} = -V_x \sin\boldsymbol{y}\cos\boldsymbol{q} - V_z(\cos\boldsymbol{y}\sin\boldsymbol{j} + \sin\boldsymbol{y}\sin\boldsymbol{q}\cos\boldsymbol{j}) + V_y(\cos\boldsymbol{y}\cos\boldsymbol{j} - \sin\boldsymbol{y}\sin\boldsymbol{q}\sin\boldsymbol{j})
\end{cases}
$$

3 Modelling of the MAV (19)

4 Control Law Design

- Linearization of the model

Suppose a ***level straight flight conditon***:

$$V_x = 20m/s, V_y = V_z = 0, \theta = \psi = \varphi = 0, \delta_e = -4^\circ, \delta_a = 0$$

The ***linear equation*** of Pégase in this flight state can be deduced:

$$\begin{cases} \dot{q} = -0.02V_x - 2.44V_z - 1.81\delta_e \\ \dot{\theta} = q \\ \dot{p} = -1.47\delta_a \\ \dot{\varphi} = p \end{cases}$$

The yaw angle can be only adjusted by the control of roll angle because only two control variables are available.Suppose that airspeed can be controlled in another close loop, the equation above becomes

$$
\begin{cases}
\dot{q} = -0.4 - 1.81\delta_e \\
\dot{\theta} = q \\
\dot{p} = -1.47\delta_a \\
\dot{\varphi} = p
\end{cases}
$$

- Controller design

Therefore a very simple attitude control law can be obtained by applying the idea of dynamic inversion:

$$\begin{cases} \boldsymbol{d}_e = -0.22 - 0.55 k_{11} [k_{12} (\boldsymbol{q}_d - \boldsymbol{q}) - q] \\ \boldsymbol{d}_a = -0.68 k_{21} [k_{22} (\boldsymbol{j}_d - \boldsymbol{j}) - p] \end{cases}$$

where $k_{11}, k_{12}, k_{21}, k_{22}$ are control parameters to guarantee enough bandwidth and q_d, j_d are desired attitude of the MAV. In fact, the close loop poles are

$$\frac{-k_{i1} \pm \sqrt{k_{i1}^2 - 4k_{i1}k_{i2}}}{2}, i = 1,2$$

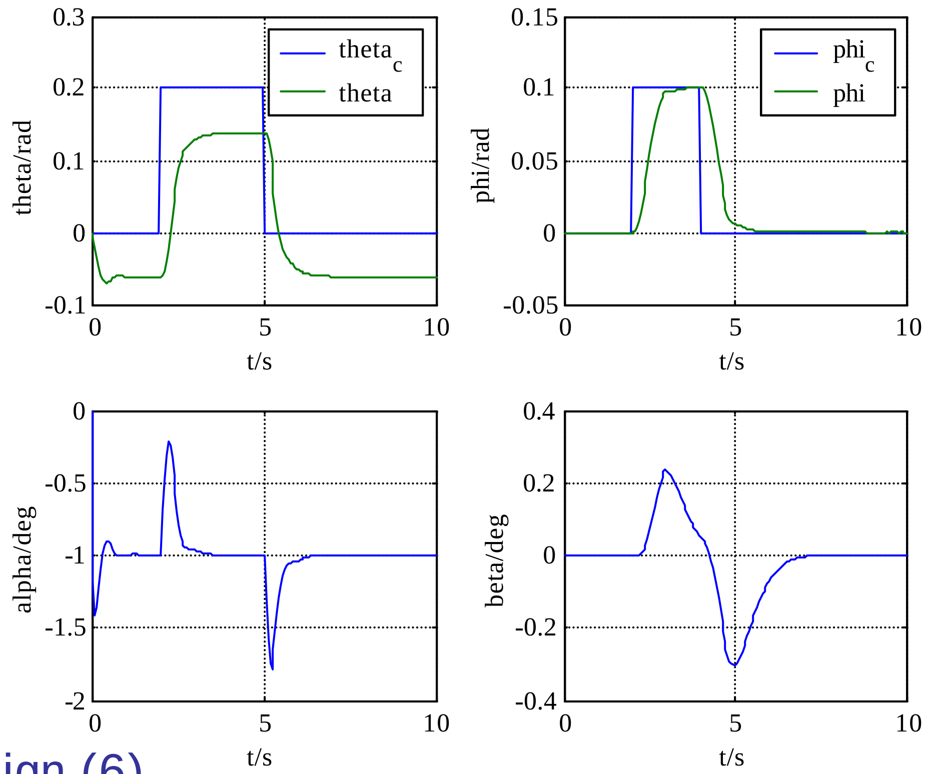
4 Control Law Design (4)

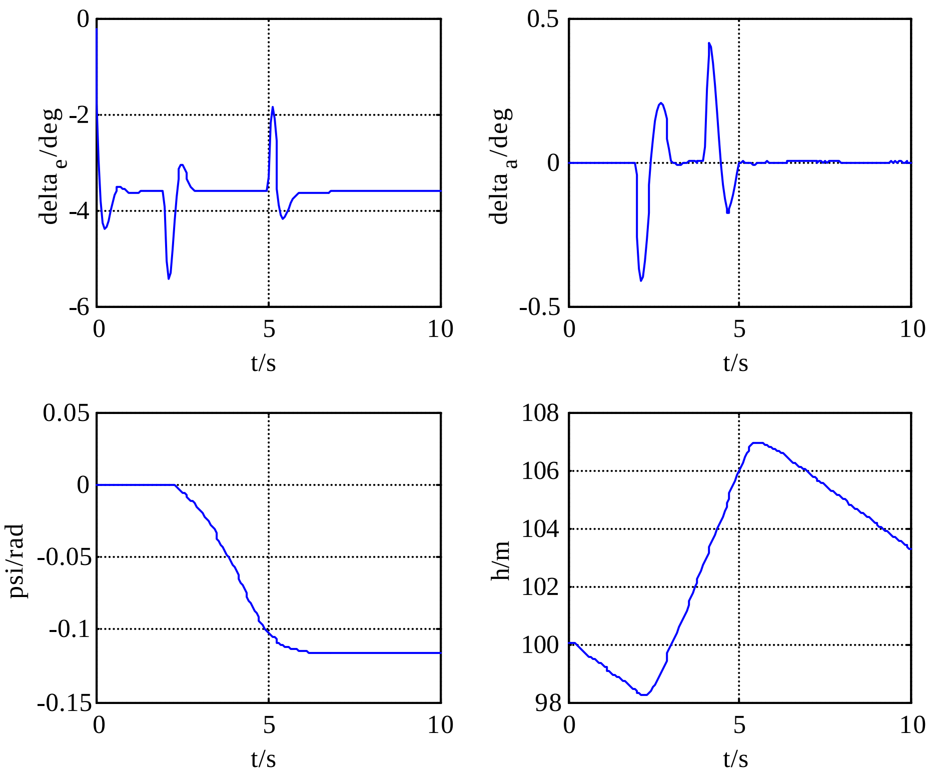
- Airspeed controller design ***(Propulsion control)***

$$Th_c = mg\sin \boldsymbol{q} + D + \mathbf{K}_{dv}{}^{T}\dot{\mathbf{V}}^{D} + \mathbf{K}_{v}{}^{T}(\mathbf{V}^{D} - \mathbf{V}_{A})$$
$$+ \mathbf{K}_{\Delta P}{}^{T}(\mathbf{P}^{D} - \mathbf{P}_{A})$$

$\mathbf{K}_{dv}, \mathbf{K}_{v}, \mathbf{K}_{\Delta P}$	are three weight vectors
$\dot{\mathbf{V}}^{D}, \mathbf{V}^{D}, \mathbf{P}^{D}$	are required acceleration, velocity and position vectors
$\mathbf{V}_{A}, \mathbf{P}_{A}$	are real velocity and position Vectors
D	is drag force

▪ Nonlinear simulation results





4 Control Law Design (7)

From the nonlinear simulation, conclusions can be drawn:

- The close loop attitude system is stable, the control law is effective
- Heading can be controled by ajusting rolling angle
- There is an error in pitching channel

5 Guidance Law Design

The guidance system includes ***altitude holding loop***, ***lateral position control*** and ***heading control*** which are based on the inner loop of attitude control. It is a kind of ***two dimensional*** guidance. The guidance algorithm is simple, effective and easy to be implemented.

- Altitude holding control

  The error of desired and actual height is introduced to the pitch angle control loop to constitute the altitude holding control loop. A saturation function is needed to limit the maximum value of feedback terms. Therefore the deflection command of elevator won.t exceed its position limit.

$$d_e = -0.22 - 0.55k_{11}[k_{12}(q_d - q) - q] + k_h sat(H - H_0)$$

- Heading control

Heading angle and the lateral distance between airplane and desired flight trajetory can be controled by ajusting rolling angle. Based on this idea, lateral guidance law can be obtained. Saturation function is used here for the same reason as in altitude holding control loop.

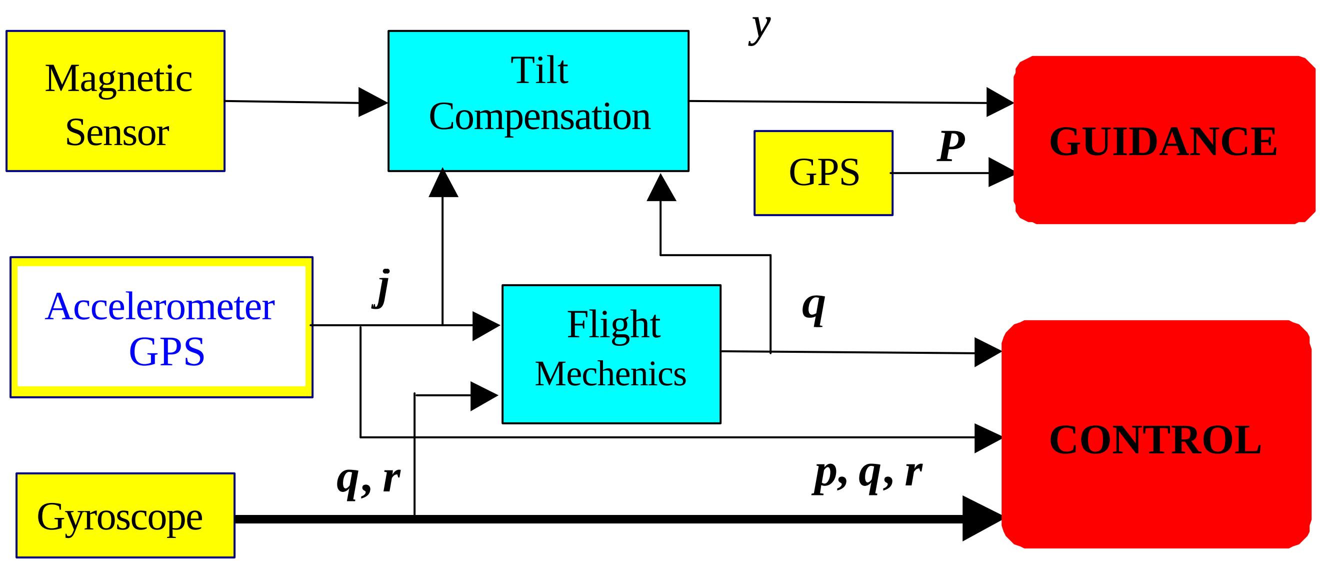
$$\boldsymbol{d}_a = -0.68k_{21}[k_{22}(\boldsymbol{j}_d - \boldsymbol{j}) - p] + k_y sat(\boldsymbol{y} - \boldsymbol{y}_d) + k_{\Delta y} sat(\Delta y)$$

5 Guidance Law Design (3)

6 Attitude Determination

To accomplish autonomous guidance and control, the following signals are needed: ***three attitude angles***, ***angular rates*** and ***real-time position***. The limited volume of the MAVs and the capability of the CPU on board prevent us from using the mature algorithm to determine the attitude. Therefore a simple enough algorithm with certain accuracy must be devised by using the sensors on board.

Basic idea to obtain attitude of MAV



7 Conclusion

- A nonlinear model of Pégase, a mini aerial vehicle, is established from the data of wind tunnel test.
- The interpolation of the force and moment coefficients is introduced detail by detail.
- A level straight flight linearization equation is obtained.
- According to this equation, based on the idea of dynamic inversion, a control law is devised and verified by nonlinear simulation.
- The guidance loop is designed.
- At last, Basic idea of attitude determination is introduced.

The whole system is effective and easy to be implemented.

Thanks!