Hovering flight stabilization in wind gusts for ducted fan UAV

Jean Michel Pflimlin Philippe Souères Tarek Hamel

***Abstract*— This paper describes a control strategy to stabilize the position of a Micro Air Vehicle in wind gusts despite unknown aerodynamic efforts. The proposed approach allows to overcome the problem of gyroscopic coupling by taking advantage from both the structure of the thrust mechanism, which is made of two counter rotating propellers, and the control strategy which involves a decoupling of the yaw rate dynamics from the rest of the system dynamics. The controller is designed by means of backstepping techniques allowing the stabilization of the vehicle's position while on-line estimating the unknown aerodynamic efforts.**

Keywords: ducted fan, micro air vehicle, UAV, backstepping control, Lyapunov control functions, zero dynamics.

I. INTRODUCTION

The design of autonomous navigation strategy for Micro Air Vehicles (MAV) has now become a very challenging research area. Those small and discreet flying vehicles, able to perform vertical takeoff and landing (VTOL), seem to be of evident interest for civil and military operations in urban environment. Autonomous VTOL vehicle capable of stationary flight have been considered in recent years. Significant research interest has been directed towards the development of autonomous scale model helicopter due to their high payload to power ratio [1], [5], [13]. Helicopters, however, are extremely dangerous in practice due to the exposed rotor blades. Very little has been done on the development of secure platforms [6], [4]. Such platforms have considerable potential for surveillance and inspection roles in dangerous or awkward environments. The small size, highly coupled dynamics and low cost of aerial robotic devices poses a number of significant challenges in both construction and control.

In this paper we discuss the dynamic modeling and control strategy for a VTOL Micro Aerial Vehicle prototype based on ducted fan technology. One of the projects of the French National Direction of Armament, DGA, is to supply the infantry with such micro-drones by 2007. As a part of this project, the groupe Bertin Technology has in charge the development of a ducted fan UAV which is represented in figure 1. The work presented in this paper has been developed in the framework of a collaboration beetween, Bertin Tecnologies, the LAAS-CNRS and the I3S Laboratory, which is devoted to the micro-drone modeling and the elaboration of autonomous control strategies.

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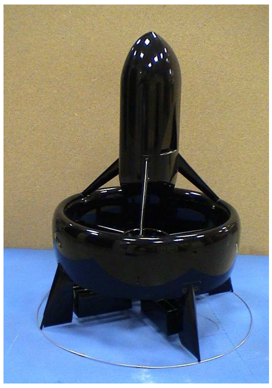

Fig. 1. A view of Bertin VTOL UAV

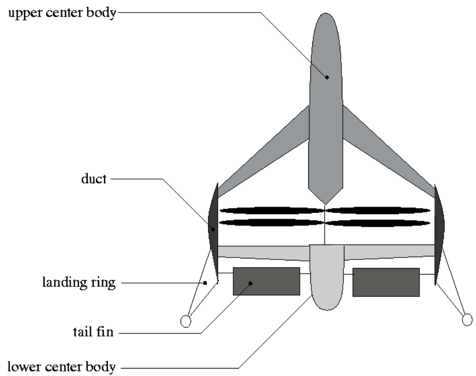

Fig. 2. Description of the Bertin VTOL UAV

This micro-drone, which is described in figure 2, must be able to perform hovering flight for surveillance applications despite possible wind perturbations. The automatic control design must allow the vehicle to be easily operated by an inexperienced user. At the end, the vehicle is expected to execute flight path, defined by sequences of navigation points, autonomously, while avoiding encountered obstacles. However, designing such autonomous navigation capabilities requires to tackle following problems:

- The aerodynamic efforts that apply on the vehicle are complex and hard to estimate and model. Indeed, due to the low Reynolds number, the atypical shape of the body, and the very wide range of angle of attack, the aerodynamic effects are strongly nonlinear and

involve flow discontinuities (stall occuring within the admissible range of angle of attack).
- The flight envelope is very wide: from hovering to cruise flight the pitch angle varies from 0 to 40 degrees. The micro-drone is able to transit from a helicopter behaviour, when standing vertically, to a plane behaviour with an annular wing executing a high-speed horizontal flight, or dash-flight, during which its velocity ranges within 100 to 150 kmph.
- This kind of vehicle is unstable and its dynamics along the three axes are strongly coupled.
- Trim control does not exist for this kind of MAV. Stabilizing the vehicle to a given attitude requires the use of control surfaces. Therefore, the ability to stabilize the vehicle in crosswind is strongly dependent on the actuator saturation problems.

In this paper we describe a control strategy to stabilize the position of the MAV in wind gusts despite unknown aerodynamic efforts. The proposed approach allows to overcome the problem of gyroscopic coupling by taking advantage from both the mechanical structure of the thrust, which is made of two counter rotating propellers, and the control strategy which involves a decoupling of the yaw rate dynamics from the rest of the system dynamics. The controller is designed by means of backstepping techniques allowing the stabilization of the MAV's position while on-line estimating the unknown aerodynamic efforts.

II. System Modeling

A. Reference frames

Two reference frames are considered to model the system (cf. [2]) :

- $\mathcal{I}$ is an inertial frame attached to the earth. It is assumed to be Galilean
- $\mathcal{A}$ is body-fixed frame attached to the vehicle. The attitude $\mathcal{R}$ of the body-fixed frame $\mathcal{A}$, with respect to the inertial frame $\mathcal{I}$, is represented by means of the Euler angles ϕ, θ, ψ.

The frames $\mathcal{I}$ and $\mathcal{A}$ are respectively associated to the vector bases $[e_1, e_2, e_3]$ and $[x_b, y_b, z_b]$. The relation between frames $\mathcal{I}$ and $\mathcal{A}$ are recalled bellow (see [8]):

$$\begin{array}{l} \mathcal{R} = [x_b, y_b, z_b]_{\mathcal{I}} \\ \mathcal{R}^{-1} = \mathcal{R}^T = [e_1, e_2, e_3]_{\mathcal{A}} \end{array} \tag{1}$$

Furthermore, denoting by $\Omega = (p, q, r)^T$ the rotation velocity of the body-fixed frame with respect to the inertial frame, expressed in $\mathcal{A}$, and representing by $\breve{\Omega}$ the skew symmetric matrix associated to Ω, we get:

$$\dot{\mathcal{R}} = \mathcal{R}\breve{\Omega} \tag{2}$$

B. Inertia matrix

The vehicle being symmetric with respect to the planes (x_b, z_b) and (y_b, z_b), the z_b-axis turns out to be a principal axis of symmetry and supports the vehicle center of gravity. With respect to the frame $\mathcal{A}$, the inertia matrix takes then the following simple form:

$$\mathbf{I} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_2 \end{pmatrix} \tag{3}$$

C. Forces acting on the system

In this section we make the simplifying hypothesis that the various aerodynamic efforts are additional. The thrust of the propellers, the lift and the drag forces generated by the air flow along the body, and the efforts of mobile surfaces are then considered separately. Experimental tests in wind tunnel will provide the global aerodynamic wrench as a function of the relative wind (aerodynamic velocity, attack and sideslip angles), the deflection angle of mobile surfaces, and the propellers rotation velocity. The aerodynamic wrench will be considered in the simulation models in order to test the robustness of the control laws. The different forces acting on the system are:

- The weight, $P = mge_3$, applied at the center of gravity G.
- The thrust of propellers, $T = -uz_b$, applied at the point F_T located on the z_b-axis. T is collinear to the lever arm vector $\overrightarrow{F_T G}$. The counter rotating propellers allow to counteract the effects of the reactive coupling on the yaw axis. As a consequence, the thrust does not generate any moment at G. The magnitude u of the thrust T constitutes the first control input of the system.
- The aerodynamic efforts, F_{ext}, are applied at the point F_a. The location of F_a is hard to determine and depends on the angle of attack. However, for symmetry reasons this point is located on the z_b-axis. Let us denote by ε the corresponding lever arm ($\overrightarrow{GF_a} = \varepsilon z_b$). Globally, F_a is located above the duct lip, the smaller the angle of attack is, the farther from the lip the point F_a is.
- The force generated by each control surface is denoted by: F_{ail_i}, $i = 1..4$. These forces induce a moment $\Gamma_{ail} = [\Gamma_l, \Gamma_m, \Gamma_n]^T$ at G and a resulting force F_{ail}.

$$F_{ail} = -\frac{1}{L} z_b \wedge \Gamma_{ail} \triangleq \Sigma\Gamma_{ail} \tag{4}$$

where L represents the lever arm of the forces generated by the control surfaces (see figure 3). Γ_{ail} constitutes the second control input of the sysyem.

Figure 3 shows the different efforts applied on the MAV.

D. Dynamic equations

The dynamic representation of the system is given by the application of the fundamental theorem of Mechanics (Newton's theorem) expressed in the inertial frame $\mathcal{I}$ and the theorem of angular momentum (Euler's theorem) ex-

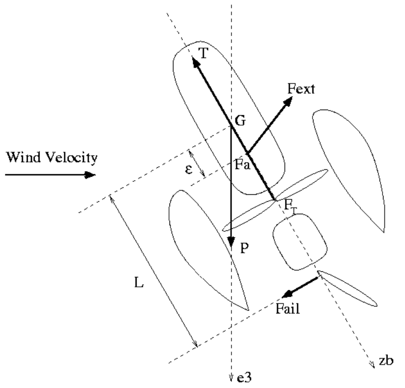

Fig. 3. Description of forces acting on the micro-drone

pressed in the body frame $\mathcal{A}$ (see [7]).

$$\begin{bmatrix} \dot{\xi} \\ m\dot{v} \\ \dot{\mathcal{R}} \\ \mathbf{I}\dot{\Omega} \end{bmatrix} = \begin{bmatrix} v \\ -u\mathcal{R}e_3 + mge_3 + \mathcal{R}\Sigma\Gamma_{ail} + F_{ext} \\ \mathcal{R}\breve{\Omega} \\ -\Omega \wedge \mathbf{I}\Omega + \Gamma_{ail} + \sigma\mathcal{R}^T F_{ext} \end{bmatrix} \quad (5)$$

The parameters $[\xi, v, \mathcal{R}, \Omega]$ and σ are defined as follows:

- ξ is the position of the center of gravity with respect to $\mathcal{I}$, expressed in the inertial base $[e_1, e_2, e_3]$,
- v is the velocity of the center of gravity with respect to $\mathcal{I}$ expressed in the inertial base $[e_1, e_2, e_3]$,
- $\mathcal{R}$ is the above-mentionned transformation matrix from $\mathcal{I}$ to $\mathcal{A}$,
- Ω is the angular velocity vector of the body frame relative to the inertial frame, expressed in the body base $[x_b, y_b, z_b]$,
- $\sigma = \varepsilon\breve{z}_b$[1]

III. Preliminary analysis of the dynamics

Interesting elements of the drone's behaviour in flight can be extracted from the dynamic equation given by (5). In a first part, we focus on the existing decoupling between the yaw dynamics and the forward flight dynamics. The problem of zero dynamics is then addressed in a second subsection.

A. Decoupling of the yaw dynamics

Thanks to the form of the inertia matrix (cf. (II-B)) , it can be shown that:

$$\Omega \wedge \mathbf{I}\Omega = \begin{pmatrix} I_2qr - I_1qr \\ I_1pr - I_2pr \\ 0 \end{pmatrix} \text{ with } \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Introducing this relation in the expression of the angular moment derivative equation, and considering only the yaw rate,'r', equation, it comes:

$$I_2\dot{r} = \Gamma_n \quad (6)$$

[1]The notation $\breve{z}_b$ denotes the skew symmetric matrix of vector z_b.

Reminding that F_a is on z_b-axis and therefore there is no aerodynamic yaw moment. Consequently, considering that $\dot{r}$ only depends on the control Γ_n, and that F_{ail} does not depend on Γ_n (see (4)), the yaw rate equation turns out to be decoupled from the other equations. Assume that $r(0) = 0$. By choosing $\Gamma_n = 0$ one ensures that $r = 0$ for $t > 0$. In practice, it is sensible to implement a control that acts to preserve this condition. Choose

$$\Gamma_n = -k_r r \quad (7)$$

where $k_r > 0$. In the rest of the paper it will be assumed that $r = 0$.

B. The problem of zero dynamics

The zero dynamics of a nonlinear system are the internal dynamics of the system subject to the constraint that the outputs (and, therefore, all derivatives of the outputs) are set to zero for all times [10], [9]. Nonlinear systems with non-asymptotically stable zero-dynamics are called strictly (or weekly, if the zero dynamics are marginally stable) non-minimum phase.

In this section, we show that the system (5), is strictly non-minimum phase and that a naive stabilization control design would result in undesirable non-decaying oscillations in the motion of the vehicle. Indeed, the coupling of the thrust T and torque control Γ_{ail} on the airframe dynamics leads to the presence of small-body-forces $\mathcal{R}\Sigma\Gamma_{ail}$ that result in zero-dynamics. The first step in analyzing the zero dynamics nature is to set to zero the outputs and all their time-derivatives for all $t \geq 0$. Assume that $F_{ext} = 0$ and consider ξ as the output vector. In this case, the translational dynamics is fully actuated[2]. Assume now that a control law stabilising ξ and v towards zero exists. In this case, the translational dynamics of (5) becomes:

$$ue_3 + \Sigma\Gamma_{ail} = mg\mathcal{R}^T e_3 \quad (8)$$

Recalling (4) and multiplying both sides of (8) by $\breve{e}_3^T$, the transpose of the skew symmetric matrix of vector e_3, it yields:

$$\pi_{e_3}\Gamma_{ail} = Lmg\breve{e}_3^T\mathcal{R}^T e_3, \ \pi_{e_3} = I_3 - e_3e_3^T$$

Substituting the above relation in the moment equation of system (5) and using the fact that the third component of Ω is null ($r = 0$), it yields the dynamic equation of an idealized spherical pendulum:

$$\pi_{e_3}\dot{\Omega} = -\frac{Lm}{I_1}g\breve{e}_3R^T e_3 \quad (9)$$

To study the stability of this dynamics, define a Lyapunov function candidate $\mathcal{V}$

$$\mathcal{V} = \frac{1}{2}\Omega^T\pi_{e_3}\mathbf{I}\pi_{e_3}\Omega + \frac{|L|m}{I_1}g(1 - e_3^TR^Te_3) \quad (10)$$

[2]This is due to the fact that the the matrix $\mathcal{R}\Sigma$ is of rank two with entries only in the first and second columns and that $\mathcal{R}e_3 \in \text{Ker}(\Sigma)$.

corresponding to the total energy of a pendulum of length $\frac{I_1}{m|L|}$. Differentiating $\mathcal{V}$, it yields:

$$\begin{aligned}\dot{\mathcal{V}} &= 0 \text{ if } L < 0,\\ &= -2\frac{Lm}{I_1} g\Omega^T \breve{e}_3 R^T e_3 \text{ if } L > 0.\end{aligned}$$

Depending on the sign of L, the zero dynamics have different qualitative behaviour. For $L < 0$, the input-output system is weekly non-minimum phase (the zero dynamics are marginally stable). Note, however, that $L > 0$ corresponds to an idealized inverted spherical pendulum. It follows from classical Lyapunov theory and the above discussion that the system is strictly non-minimum phase.

The first contribution of the paper is to show that, deporting the control point away from the center of masse of the vehicle, in the opposite direction to the mobile surfaces location, can lead to cancel the zero dynamics effects inherent in the system. More precisely, let ξ_D be the new control point:

$$\xi_D = \xi + \overrightarrow{GD}, \tag{11}$$

Then, if $\overrightarrow{GD}$ is a translation vector along the z_b axis defined by:

$$\overrightarrow{GD} = dz_b \quad \text{with} \quad d = -\frac{I_1}{mL}, \tag{12}$$

The dynamics of system (5) becomes:

$$\begin{cases} \dot{\xi_D} = v_D \\ m\dot{v_D} = -\bar{u}\mathcal{R}e_3 + mge_3 + (I_{3\times3} - md\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\sigma\mathcal{R}^T)F_{ext} \\ \dot{\mathcal{R}} = \mathcal{R}\breve{\Omega} \\ \mathbf{I}\dot{\Omega} = \Gamma_{ail} + \sigma\mathcal{R}^T F_{ext} \end{cases} \tag{13}$$

where the zero dynamics do not appear any more and:

$$\begin{gathered}\xi_D = \xi + d\mathcal{R}e_3\\ v_D = v + d\mathcal{R}\breve{\Omega}e_3\\ \bar{u} = u + md(p^2 + q^2)\end{gathered}$$

In the next section, the above system will be used to develop the control design for the stabilization task in presence of perturbation forces (constant gusts).

IV. Control design

In our modeling, there are coarsely two sources of incertainty : the components of aerodynamic forces applied to the body, and the point of application of those efforts. More precisely, in model (13), F_{ext} and σ are unknown. The main difficulties comes from the term $md\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\sigma\mathcal{R}^T$ in the tranlational dynamics and the bilinear form $\sigma\mathcal{R}^T F_{ext}$ in the moment equations. Both difficulties can be overcome with the following considerations:

- As $\|\mathcal{R}\| = 1$ and $\|\breve{z}_b\| = 1$, the magnitude of the perturbating term $md\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\sigma\mathcal{R}^T$ is strongly dependent on the ratio $\frac{md}{I_1}||\sigma|| = \frac{md}{I_1}\varepsilon$. Taking into account the expression of d given by (12), this ratio is equivalent to $\frac{\varepsilon}{L}$, i.e. the ratio between the lever arm of the aerodynamic forces and the lever arm of the control surfaces. For this kind of sit-on-tail VTOL this ratio is usually weak ($\frac{\varepsilon}{L} << 1$). Therefore, the approach taken in our control algorithm, consists in designing a robust controller for the system without considering the perturbation term and analyze the robustness of the closed loop with respect to the perturbation term. For the sake of clarity of the presentation, we address the robustness problem by analyzing some simulation results and leave further discussion on dynamic perturbation in a further publication.
- Using the expression of σ (see (II-D)), $\sigma\mathcal{R}^T F_{ext}$ can be written as: $\varepsilon\breve{z}_b\mathcal{R}^T F_{ext}$, i.e., as ε is a scalar: $\breve{z}_b\mathcal{R}^T\varepsilon F_{ext}$. At this stage, the unknown vector M_{ext}, homogeneous to a moment, is defined as follows:

$$M_{ext} = \varepsilon F_{ext}$$

Finally, both unknown vectors F_{ext} and M_{ext} are taken into account in the control design[3].Those considerations being assumed, the model used for the control design is the following one:

$$\begin{bmatrix} \dot{\xi_D} \\ m\dot{v_D} \\ \dot{\mathcal{R}} \\ \mathbf{I}\dot{\Omega} \end{bmatrix} = \begin{bmatrix} v_D \\ -\bar{u}\mathcal{R}e_3 + mge_3 + F_{ext} \\ \mathcal{R}\breve{\Omega} \\ \Gamma_{ail} + \breve{z}_b\mathcal{R}^T M_{ext} \end{bmatrix} \tag{14}$$

The backstepping technique is used to design a control law which adapts to incertainties F_{ext} and M_{ext}.

Step 1. Let ξ_s be a constant desired position for the control point D. Define $\delta_1 = \xi_D - \xi_s$ as the error between the actual and desired position of the control point D of the MAV. The control law must ensure the convergence of the error δ_1 towards zero. Therefore, define S_1 as a storage function associated with the error term δ_1:

$$S_1 = \frac{1}{2}\delta_1^T\delta_1$$

Differentiating S_1, it yields:

$$\dot{S}_1 = \delta_1^T\dot{\delta}_1$$

Introducing a gain $k_1 > 0$, recalling equation (14) and differentiating δ_1, it yields:

$$\dot{\delta}_1 = \dot{\xi}_D = v_D = -k_1\delta_1 + \underbrace{k_1\delta_1 + v_D}_{\frac{\delta_2}{m}}$$

Consequently, at the end of the first step of backstepping, it comes :

$$\begin{aligned}\dot{S}_1 &= -k_1|\delta_1|^2 + \tfrac{1}{m}\delta_1^T\delta_2\\ \dot{\delta}_1 &= -k_1\delta_1 + \tfrac{1}{m}\delta_2\end{aligned} \tag{15}$$

Step 2. δ_2 has been introduced, in the expression of $\dot{\delta}_1$, as the difference between the real velocity of D and a virtual control which would lead to an exponential convergence of δ_1, when $\delta_2 = 0$. To stabilize the system, the convergence of

[3] keep in mind that M_{ext} does not represent the aerodynamic moment. It is only a vector colinear to F_{ext}

δ_2 must be ensured. In the expression of the time derivative $\dot{\delta}_2$, the unknown vector F_{ext} appears via the translation dynamics. This unknown vector will be estimated in the control law by $\hat{F}_{ext}$. The convergence of the error $\tilde{F}_{ext}$ between the real aerodynamic forces and the estimated ones shall be ensured too. Therefore, a second Lyapunov function S_2 is defined as follows:

$$S_2 = S_1 + \frac{1}{2}\delta_2^T\delta_2 + \frac{1}{2}\tilde{F}_{ext}^T\Gamma_F^{-1}\tilde{F}_{ext}$$

where $\delta_2 = mv_D + mk_1\delta_1$ and $\tilde{F}_{ext} = F_{ext} - \hat{F}_{ext}$. In the expression of S_2, Γ_F is a scalar gain which tunes up the dynamics of the adaption of F_{ext}. Using the expression of $\dot{S}_1$ given by (15), it comes:

$$\dot{S}_2 = -k_1|\delta_1|^2 + \frac{1}{m}\delta_1^T\delta_2 + \delta_2^T\dot{\delta}_2 - \tilde{F}_{ext}^T\Gamma_F^{-1}\dot{\hat{F}}_{ext}$$

Introducing the gain $k_2 > 0$, the time derivative $\dot{\delta}_2$ is given by:

$$\begin{aligned} \dot{\delta}_2 &= mk_1\dot{\delta}_1 + m\dot{v}_D \\ &= -mk_1^2\delta_1 + k_1\delta_2 - \bar{u}\mathcal{R}e_3 + mge_3 + F_{ext} \\ &= -mk_1^2\delta_1 - k_2\delta_2 + (k_1+k_2)\delta_2 - \bar{u}\mathcal{R}e_3 \\ &\quad +mge_3 + \hat{F}_{ext} + \tilde{F}_{ext} \end{aligned} \tag{16}$$

Using (16) in the expression of $\dot{S}_2$, and gathering the terms[4] $\delta_1^T\delta_2$ and $\tilde{F}_{ext}$, it comes:

$$\begin{aligned} \dot{S}_2 &= -k_1|\delta_1|^2 - k_2|\delta_2|^2 + (\tfrac{1}{m} - mk_1^2)\delta_1^T\delta_2 \\ &\quad +\delta_2^T\big((k_1+k_2)\delta_2 - \bar{u}\mathcal{R}e_3 + mge_3 + \hat{F}_{ext}\big) \\ &\quad +\tilde{F}_{ext}^T(\delta_2 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) \end{aligned}$$

At this stage, let us introduce:

$$\begin{cases} \tau_1 = \delta_2 \\ \alpha_1 = (\frac{1}{m} - mk_1^2)\delta_1 + (k_1+k_2)\delta_2 + mge_3 + \hat{F}_{ext} \end{cases} \tag{17}$$

Then it can be shown that $\dot{S}_2$ exresses as:

$$\dot{S}_2 = -k_1|\delta_1|^2 - k_2|\delta_2|^2 + \delta_2^T\underbrace{(\alpha_1 - \bar{u}\mathcal{R}e_3)}_{-\delta_3} + \tilde{F}_{ext}^T(\tau_1 - \Gamma_F^{-1}\dot{\hat{F}}_{ext})$$

If $-\bar{u}\mathcal{R}e_3$ was a control vector, the adaptive filter τ_1 and the control law α_1 would cancel the influence of $\tilde{F}_{ext}$ and guarantee the non positivity of $\dot{S}_2$. However, if $\bar{u}$ is really a control input, the orientation of the thrust $\mathcal{R}e_3$ is not. Therefore, the difference δ_3 between the real thrust $\bar{u}\mathcal{R}e_3$ and the virtual thrust α_1 is introduced. Convergence of δ_3 must be ensured. At the end of the second step of backstepping, it comes:

$$\begin{aligned} \dot{S}_2 &= -k_1|\delta_1|^2 - k_2|\delta_2|^2 - \delta_2^T\delta_3 + \tilde{F}_{ext}^T(\tau_1 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) \\ \dot{\delta}_2 &= -\tfrac{1}{m}\delta_1 - k_2\delta_2 - \delta_3 + \tilde{F}_{ext} \end{aligned} \tag{18}$$

Step 3. Let us introduce now the third Lyapunov function S_3 :

$$S_3 = S_2 + \tfrac{1}{2}\delta_3^T\delta_3 \quad \text{with} \quad \delta_3 = \bar{u}\mathcal{R}e_3 - \alpha_1$$

[4] Let us recall that, terms in Lyapunov functions being scalar, it comes $\delta_2^T\tilde{F}_{ext} = \tilde{F}_{ext}^T\delta_2$

Using the expression of $\dot{S}_2$ given by (18), it can be deducted that:

$$\dot{S}_3 = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - \delta_2^T\delta_3 + \tilde{F}_{ext}^T(\tau_1 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) + \delta_3^T\dot{\delta}_3$$

With $\dot{\delta}_3$ given by:

$$\dot{\delta}_3 = \dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\dot{\mathcal{R}}e_3 - \dot{\alpha}_1 = \dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 - \dot{\alpha}_1$$

Expressions of $\dot{\delta}_1$ and $\dot{\delta}_2$ given by (15) and (18) respectively lead to the following formulation of $\dot{\alpha}_1$:

$$\begin{aligned} \dot{\alpha}_1 = &\ (\tfrac{1}{m} - mk_1^2)\dot{\delta}_1 + (k_1+k_2)\dot{\delta}_2 + \dot{\hat{F}}_{ext} \\ = &\ (\tfrac{1}{m} - mk_1^2)(-k_1\delta_1 + \tfrac{1}{m}\delta_2) \\ &+(k_1+k_2)(-\tfrac{1}{m}\delta_1 - k_2\delta_2 - \delta_3 + \tilde{F}_{ext}) + \dot{\hat{F}}_{ext} \end{aligned}$$

Introducing :

$$\begin{aligned} K_1 &= -k_1(\tfrac{2}{m} - mk_1^2) - \tfrac{k_2}{m} \\ K_2 &= \tfrac{1}{m^2} - k_1^2 - k_1k_2 - k_2^2 \end{aligned}$$

Final expression of $\dot{\delta}_3$ is given as follows:

$$\begin{aligned} \dot{\delta}_3 = &\ \dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 - K_1\delta_1 - K_2\delta_2 \\ &+(k_1+k_2)\delta_3 - \dot{\hat{F}}_{ext} - (k_1+k_2)\tilde{F}_{ext} \end{aligned}$$

Reporting this expression in $\dot{S}_3$, it comes:

$$\begin{aligned} \dot{S}_3 = &\ -k_1|\delta_1|^2 - k_2|\delta_2|^2 - \delta_2^T\delta_3 + (k_1+k_2)|\delta_3|^2 \\ &+\delta_3^T(\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 - K_1\delta_1 - K_2\delta_2 - \dot{\hat{F}}_{ext}) \\ &+\tilde{F}_{ext}^T(\tau_1 - (k_1+k_2)\delta_3 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) \end{aligned}$$

Introducing the gain $k_3 > 0$, the following functions are defined:

$$\begin{cases} \tau_2 = & \tau_1 - (k_1+k_2)\delta_3 \\ \alpha_2 = & K_1\delta_1 + (1+K_2)\delta_2 - (k_1+k_2+k_3)\delta_3 + \Gamma_F\tau_2 \end{cases} \tag{19}$$

It can be easily verified that $\dot{S}_3$ expresses as:

$$\begin{aligned} \dot{S}_3 = &\ -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 \\ &+\tilde{F}_{ext}^T(\tau_2 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) + \delta_3^T(\Gamma_F\tau_2 - \dot{\hat{F}}_{ext}) \\ &+\delta_3^T\underbrace{(\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 - \alpha_2)}_{\delta_4} \end{aligned}$$

The explicit form of $\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3$ is given by:

$$\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 = \mathcal{R}\begin{bmatrix} \bar{u}q \\ -\bar{u}p \\ \dot{\bar{u}} \end{bmatrix} \tag{20}$$

Therefore, if the roll rate and the pitch rate were control inputs, the adaptive filter τ_2 and the control law on p, q et $\dot{\bar{u}}$ defined by :

$$\begin{bmatrix} \bar{u}q \\ -\bar{u}p \\ \dot{\bar{u}} \end{bmatrix} = \mathcal{R}^T\alpha_2$$

would ensure the non-positivity of $\dot{S}_3$. However, Ω is still not a control input, and a new gap δ_4 between the virtual control α_2 and the real vector $\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3$ has to be defined. At the end of the third step of the backstepping, it comes:

$$\begin{array}{rl} \dot{S}_3 = & -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 + \delta_3^T\delta_4 \\ & +\tilde{F}_{ext}^T(\tau_2 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) + \delta_3^T(\Gamma_F\tau_2 - \dot{\hat{F}}_{ext}) \\ \dot{\delta}_3 = & \delta_2 - k_3\delta_3 + \delta_4 - (k_1+k_2)\tilde{F}_{ext} + \Gamma_F\tau_2 - \dot{\hat{F}}_{ext} \end{array} \tag{21}$$

Step 4. In the expression of the time derivative $\dot{\delta}_4$, appears the angular velocity dynamics, perturbated by the second unknown parameter M_{ext}. In the control law, this term will be estimated by $\hat{M}_{ext}$. To ensure the convergence of the estimation error $\tilde{M}_{ext}$, the following Lyapunov function S_4 is defined:

$$S_4 = S_3 + \frac{1}{2}\delta_4^T\delta_4 + \frac{1}{2}\tilde{M}_{ext}^T\Gamma_M^{-1}\tilde{M}_{ext}$$

with $\delta_4 = \dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3 - \alpha_2$ and $\tilde{M}_{ext} = M_{ext} - \hat{M}_{ext}$

Γ_F is a scalar gain which tunes up the dynamics of the adaption of M_{ext} . It can be deducted that:

$$\dot{S}_4 = \dot{S}_3 + \delta_4^T\dot{\delta}_4 - \tilde{M}_{ext}^T\Gamma_M^{-1}\dot{\hat{M}}_{ext}$$

where $\dot{\delta}_4$ is given by:

$$\dot{\delta}_4 = \quad \ddot{\bar{u}}\mathcal{R}e_3 + 2\dot{\bar{u}}\mathcal{R}\breve{\Omega}e_3 + \bar{u}\mathcal{R}\breve{\Omega}^2e_3 + \bar{u}\mathcal{R}\dot{\breve{\Omega}}e_3 - \dot{\alpha}_2$$

Noticing that:

$$\mathcal{R}\tilde{\Omega}^2e_3 = \mathcal{R}\begin{bmatrix} -q^2 & qp & 0 \\ qp & -p^2 & 0 \\ 0 & 0 & -(p^2+q^2) \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -(p^2+q^2)\mathcal{R}e_3$$

Then $\dot{\delta}_4$ expresses as[5]:

$$\begin{array}{rl} \dot{\delta}_4 = & \left[\ddot{\bar{u}} - \bar{u}(p^2+q^2)\right]\mathcal{R}e_3 + 2\dot{\bar{u}}\mathcal{R}\breve{\Omega}e_3 - \bar{u}\mathcal{R}\breve{e}_3\dot{\Omega} - \dot{\alpha}_2 \\ = & \left[\ddot{\bar{u}} - \bar{u}(p^2+q^2)\right]\mathcal{R}e_3 + 2\dot{\bar{u}}\mathcal{R}\breve{\Omega}e_3 \\ & -\bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}(\Gamma_{ail} + \breve{z}_b\mathcal{R}^TM_{ext}) - \dot{\alpha}_2 \end{array}$$

Expressing τ_2 with respect to δ_2 and δ_3 (see (17) and (19)), the time derivative $\dot{\alpha}_2$ can be written as follows:

$$\begin{array}{rcl} \dot{\alpha}_2 & = & K_1\dot{\delta}_1 + (1+K_2)\dot{\delta}_2 - (k_1+k_2+k_3)\dot{\delta}_3 \\ & & +\Gamma_F(\dot{\delta}_2 - (k_1+k_2)\dot{\delta}_3) \\ & = & K_1\dot{\delta}_1 + (1+K_2+\Gamma_F)\dot{\delta}_2 \\ & & -\big((k_1+k_2)(1+\Gamma_F)+k_3\big)\dot{\delta}_3 \end{array}$$

Reporting in $\dot{\alpha}_2$ the expression of $\dot{\delta}_i$, $i = 1..3$ given by (15), (18) and (21) respectively, and gathering the terms, it can be shown that:

$$\dot{\alpha}_2 = G_1\delta_1 + G_2\delta_2 + G_3\delta_3 + G_4\tilde{F}_{ext} + G_5(\dot{\hat{F}}_{ext} - \Gamma_F\tau_2 - \delta_4)$$

with

$$\left\{\begin{array}{rcl} G_1 & = & -k_1K_1 - \frac{1}{m}(K_2+1+\Gamma_F) \\ G_2 & = & \frac{K_1}{m} - k_2(K_2+1+\Gamma_F) - G_5 \\ G_3 & = & -(K_2+1+\Gamma_F) + G_5k_3 \\ G_4 & = & K_2+1+\Gamma_F + G_5(k_1+k_2) \\ G_5 & = & (k_1+k_2)(1+\Gamma_F) + k_3 \end{array}\right.$$

[5]Recall that for any vector u, v: $\breve{u}v = u \times v = -v \times u = -\breve{v}u$

Finally, isolating the estimation errors $\tilde{F}_{ext}$ and $\tilde{M}_{ext}$, the following expression of $\dot{\delta}_4$ is obtained:

$$\begin{array}{l} \dot{\delta}_4 = \ddot{\bar{u}}\mathcal{R}e_3 - \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\Gamma_{ail} \\ \quad -\bar{u}(p^2+q^2)\mathcal{R}e_3 + 2\dot{\bar{u}}\mathcal{R}\breve{\Omega}e_3 - \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T\hat{M}_{ext} \\ \quad -G_1\delta_1 - G_2\delta_2 - G_3\delta_3 - G_5(\dot{\hat{F}}_{ext} - \Gamma_F\tau_2 - \delta_4) \\ \quad -\bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T\tilde{M}_{ext} - G_4\tilde{F}_{ext} \end{array} \tag{22}$$

In equation (22), the terms are arranged in such way that: the first line is the control vector input, the second and third lines gather “measurable” terms (internal states of the system and estimations of the unknown vectors) which can be integrated directly in a control law, and the last line contains the estimation errors of the unknown parameters. At this stage, introducing $k_4 > 0$, the following functions are defined:

$$\left\{\begin{array}{rcl} \tau_3 & = & \tau_2 - G_4\delta_4 \\ \alpha_3 & = & \delta_3 + (k_4+G_5)\delta_4 - \bar{u}(p^2+q^2)\mathcal{R}e_3 \\ & & +2\dot{\bar{u}}\mathcal{R}\breve{\Omega}e_3 - \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T\hat{M}_{ext} \\ & & -G_1\delta_1 - G_2\delta_2 - G_3\delta_3 - G_5\Gamma_F(\tau_3 - \tau_2) \end{array}\right. \tag{23}$$

Furthermore, the explicit form of the vector $\ddot{\bar{u}}\mathcal{R}e_3 - \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\Gamma_{ail}$ is given by:

$$\ddot{\bar{u}}\mathcal{R}e_3 - \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\Gamma_{ail} = \mathcal{R}\begin{bmatrix} \frac{\bar{u}}{I_1}\Gamma_m \\ -\frac{\bar{u}}{I_1}\Gamma_l \\ \ddot{\bar{u}} \end{bmatrix}$$

The components of this vector are clearly control inputs of the system. Introducing the ν function, that will be identified later, the control law is given by the following expression which is well defined so long as the thrust $\bar{u}$ does not vanish:

$$\begin{bmatrix} \frac{\bar{u}}{I_1}\Gamma_m \\ -\frac{\bar{u}}{I_1}\Gamma_l \\ \ddot{\bar{u}} \end{bmatrix} = \mathcal{R}^T(-\alpha_3 + \nu) \tag{24}$$

Using this control law, $\dot{S}_4$ expresses as:

$$\begin{array}{rcl} \dot{S}_4 & = & -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2 \\ & & +\delta_4^T\nu + G_5\delta_4^T(\Gamma_F\tau_3 - \dot{\hat{F}}_{ext}) + \delta_3^T(\Gamma_F\tau_2 - \dot{\hat{F}}_{ext}) \\ & & +\tilde{F}_{ext}^T(\tau_3 - \Gamma_F^{-1}\dot{\hat{F}}_{ext}) \\ & & -\tilde{M}_{ext}^T\left[(\bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T)^T\delta_4 + \Gamma_M^{-1}\dot{\hat{M}}_{ext}\right] \end{array}$$

At this stage, adaptive filters are chosen to cancel the action of $\tilde{F}_{ext}$ and $\tilde{M}_{ext}$ in the expression of $\dot{S}_4$:

$$\begin{array}{rcl} \dot{\hat{F}}_{ext} & = & \Gamma_F\tau_3 \\ \dot{\hat{M}}_{ext} & = & -\Gamma_M(\bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T)^T\delta_4 \end{array} \tag{25}$$

Assuming this choice of adaptive filters, the following expression of $\dot{S}_4$ can be deduced:

$$\begin{array}{rcl} \dot{S}_4 & = & -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2 \\ & & +\delta_4^T\nu + \delta_3^T\Gamma_F(\tau_2 - \tau_3) \end{array}$$

Using (23), it is clear that:

$$\tau_2 - \tau_3 = G_4\delta_4$$

To ensure the non positivity of $\dot{S}_4$, ν is naturally defined as follows:

$$\nu = -G_4 \Gamma_F \delta_3 \tag{26}$$

At the end of backstepping process, an adaptive control Lyapunov function has been built, and its time derivative is non-positive:

$$\dot{S}_4 = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2 \tag{27}$$

To achieve the control design, the control law Γ_n defined by equation (6), which ensures the stabilization of the yaw rate dynamics, must be added. Let us introduce the ultimate Lyapunov function S_5 :

$$S_5 = S_4 + \frac{1}{2}r^2$$

Using (27), and (6), the derivative $\dot{S}_5$ expresses as follows:

$$\dot{S}_5 = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2 + r\frac{\Gamma n}{I_2}$$

and can be finally written:

$$\dot{S}_5 = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2 - k_5 r^2 \tag{28}$$

Theorem 1: The system (14) is globally adaptively stabilizable by the control law given by (24) and by the yaw moment control law defined by (7). Moreover, the adaptive filters chosen in (25) ensure the convergence of the estimated unknown vectors to their real value. More precisely,

$$\begin{array}{rcl} \xi_D & \longrightarrow & \xi_s \\ \hat{F}_{ext} & \longrightarrow & F_{ext} \\ \tilde{M}_{ext} & \longrightarrow & 0 \\ r & \longrightarrow & 0 \end{array}$$

Proof. Equation (28) implies that δ_1, δ_2 δ_3, δ_4, r and their successive time-derivative converge to zero. If δ_1 tends to zero, then ξ_D tends to ξ_s. In the same way, the convergence of $\dot{\delta}_1$ to zero means the convergence of v_D to zero. The expression of $\dot{\delta}_2$ given by (18) implies that $\tilde{F}_{ext}$ goes to zero, too. The convergence of δ_3 indicates that the thrust direction is tilted to counteract the aerodynamic forces: $\bar{u}\mathcal{R}e_3 \longrightarrow mge_3 + F_{ext}$. The controller will adapt the thrust intensity in order to maintain the lift ($\bar{u} \longrightarrow \|mge_3 + F_{ext}\|$). The expression of α_2 given by (19) shows that $\dot{\bar{u}}\mathcal{R}e_3 + \bar{u}\mathcal{R}\breve{\Omega}e_3$ tends to zero. It means (cf. the explicit form of this vector given by (20)) that Ω converges to zero, as $\dot{\bar{u}}$ does. Finally, taking into account the control law given by (24), $\dot{\delta}_4$ expresses as:

$$\begin{array}{rcl} \dot{\delta}_4 & = & -(1+\Gamma_F G_4)\delta_3 - k_4\delta_4 - G_4\tilde{F}_{ext} \\ & & -(\Gamma_M \bar{u}\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\breve{z}_b\mathcal{R}^T)\tilde{M}_{ext} \end{array}$$

Its convergence naturally implies the convergence of $\tilde{M}_{ext}$ to zero. □

V. SIMULATION RESULTS

Simulations were performed on the base of model (13), including the perturbating term $md\mathcal{R}\breve{e}_3\mathbf{I}^{-1}\sigma\mathcal{R}^T$ neglected in the control design. The figure 4 shows the evolution of vehicle's position and attitude during the stabilization towards a fixed desired position, in presence of constant wind gusts. On-line estimation of the unknown parameters is also shown: the graph of F_{ext} with respect to time represents the evolution of $\hat{F}_{ext}$ components (dashed line) and the corresponding vector obtained from the orientation dynamics $\frac{1}{\varepsilon}\hat{M}_{ext}$ (solid line). Due to the perturbating term, the relation $\hat{F}_{ext} = \frac{1}{\varepsilon}\hat{M}_{ext}$ is lost. However, the adaptive controller succeeds in reaching the desired position. Figure 5 shows the vehicle flying behaviour when the on-line estimation of unknown parameters is deactivated: the vehicle is drifted away by crosswinds, and get stabilized to an equilibrium point deported from the expected position.

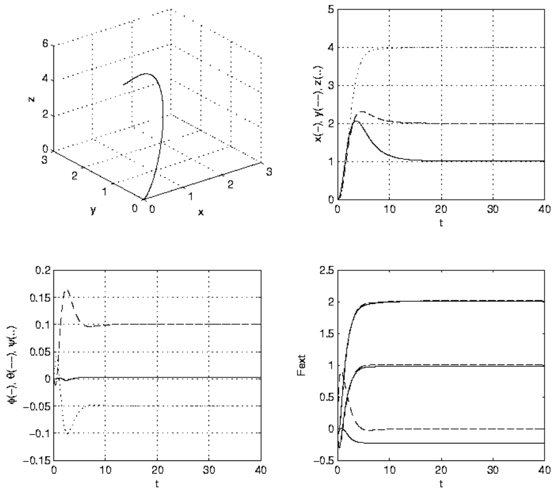

Fig. 4. Flying behaviour of the UAV in crosswind with the adaptive controller

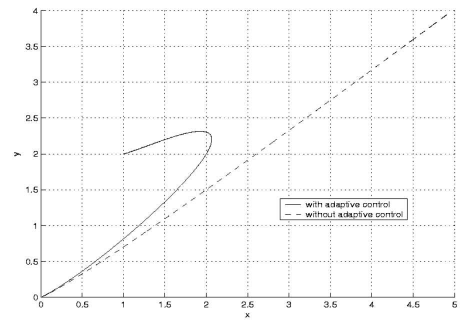

Fig. 5. Stabilization of the vehicle's position with and without on-line estimation of unknown parameters: starting position $(0, 0)$; desired final position $(1, 2)$.

VI. Concluding remarks

In this draft, we have proposed a simple model for the dynamics of a VTOL Micro Aerial Vehicle prototype that includes certain important aerodynamics effects. The unknown aerodynamical efforts are considered to be constant (or quasi-constant). An adaptive control design allowing the stabilization of the MAV to a given position is proposed. A theorem is given proving the convergence of the estimation errors to zero and simulation results are provided to illustrate the proposed concept.

In a further work, a modification of the proposed design will be undertaken to improve robustness of the closed loop design with respect to the ignored term and fluctuation in the aerodynamical efforts. To improve robustness of the adaptation process and to insure practical stability of MAV in a small compact domain around the desired position, we will add an additional σ-modification to the estimator dynamics.

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