A UAV FOR BRIDGES' INSPECTION : VISUAL SERVOING CONTROL LAW WITH ORIENTATION LIMITS

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Abstract: This paper describes the dynamics of an Unmanned Aerial Vehicle (UAV) for monitoring of structures and maintenance of bridges. It presents a novel control law based on computer vision for quasi-stationary flights above a planar target. The first part of the UAV's mission is the navigation from an initial position to a final position in an unknown 3D environment. The new control law uses the homography matrix computed from the information coming from the vision system. It will be derived with backstepping techniques. In order to keep the target in the camera's field of view, the control law uses saturation functions for bounding the UAV orientation and limiting it to very small values.

Keywords: Space and aerial robots, Vision based navigation, Guidance and control.

1. INTRODUCTION

In the past few years, a great interest in unmanned aerial vehicles has risen in military applications as well as civil applications. Their highly coupled dynamics and their small size provide a test ground for complex control theories and autonomous navigation.

In LCPC-Paris, we have recently started a project called PMI (Instrumentation Plate-Form) which is a UAV capable of quasi-stationary flights whose mission is the inspection of bridges and localization of defects and cracks. All bridges must be inspected in details every 4 to 5 years. With flying vehicles, inspection will be more secure and less expensive by reducing the number of workers, avoiding the use of footbridges (figure 1) and not obstructing the circulation traffic.

Almost all control theories for UAV's are built around a vision system, using visual servoing as a control method (Hamel and Mahony, 2000; Shell and Dickmanns, 1994). A typical vision system will include an off-the-shelf camera, an Inertial Navigation System (INS) and in some cases a Global Positioning System (GPS).

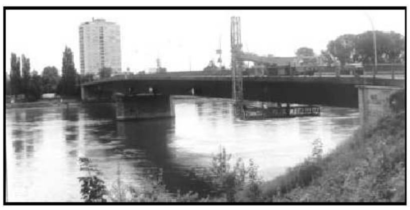

Fig. 1. Footbridge for crack's inspection

How should the information from vision sensors be used for robotic control purposes? There exists three different methods of Visual Servoing: 3D, 2D and $2\frac{1}{2}$D. 3D Visual Servoing lead to a cartesian motion planning problem. Its main drawback is the need of a perfect knowledge of the target geometric model. The second class known as 2D Visual Servoing aims to control the dynamics of features in the image plane directly (Hutchinson *et al.*, 1996). Classical 2D methods suffer from the high coupling dynamics between translation and rotational motion which makes the cartesian trajectory uncontrollable. In this paper we use a third method presented in (Malis, 1998) ($2\frac{1}{2}$D Visual Servoing) that consists of combining visual features obtained directly from the image, and features expressed in the Euclidean space. More precisely, a homography matrix is estimated from the planar feature points extracted from the two images (corresponding to the current and desired poses). From the homography matrix, we will estimate the relative position of the two views.

In this paper, we consider a general mechanical model of a flying robot capable of quasi-stationary maneuvers. Then we derive a control law from classical backstepping techniques (Hamel and Mahony, 2000) of autonomous hovering systems based on separating the translational from the rotational rigid body (airframe) dynamics. A novel approach is also presented, it will limit the orientation of the UAV. Limiting the orientation will ensure that the object will remain in the camera's field of view. We will prove the stability of such a strategy based on saturation functions. Lastly, we present simulation results of the new control law.

2. A GENERAL UAV DYNAMIC MODEL

To derive a general dynamic model for a UAV is not an easy task because each model has its own capabilities and aerodynamical properties. In this section, we will derive mechanical equations for UAV's in hover or quasi-stationary conditions.

Let $F^* = \{E_x, E_y, E_z\}$ denote a right-hand inertial or world frame such that E_z denotes the vertical direction downwards into the earth. Let $\xi = (x, y, z)$ denote the position of the centre of mass of the object in the frame F^* relative to a fixed origin in F^*. Let $F = \{E_1^a, E_2^a, E_3^a\}$ be a (right-hand) body fixed frame for the airframe. The orientation of the airframe is given by a rotation $R : F \to F^*$, where $R \in SO(3)$ is an orthogonal rotation matrix.

Let $V \in F$ denote the linear velocity and $\Omega \in F$ denote the angular velocity of the airframe both expressed in the body fixed frame. Let m denote the mass of the rigid object and let $\mathbf{I} \in \Re^{3\times 3}$ be the constant inertia matrix around the centre of mass (expressed in the body fixed frame F). Newton's equations of motion yield the following dynamic model for the motion of a rigid object:

$$\dot{\xi} = RV \tag{1}$$
$$m\dot{V} = -m\Omega \times V + \mathcal{F} \tag{2}$$
$$\dot{R} = R\,\mathrm{sk}(\Omega), \tag{3}$$
$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \Gamma. \tag{4}$$

where $\mathcal{F}$ is the vector forces and Γ is the vector torques.The notation $\mathrm{sk}(\Omega)$ denotes the skew-symmetric matrix such that $\mathrm{sk}(\Omega)v = \Omega \times v$ for the vector cross-product $\times$ and any vector $v \in \Re^3$. The vector force $\mathcal{F}$ is defined as follows :

$$\mathcal{F} = mgR^T e_3 - ue_3$$

In the above notation, g is the acceleration due to gravity, and u represents the thrust magnitude.

3. CAMERA MODELING AND VISUAL SERVOING METHOD

In this section we will present a brief discussion of the camera projection model and then introduce the homography to use the $2\frac{1}{2}$D Visual Servoing method.

3.1 Projection Model and Planar Homography

Visual data is obtained via a projection of real world images onto the camera image surface. The pose of the camera determines a rigid body transformation from the world or inertial frame F^* to the camera fixed frame F (and vice-versa). One has

$$P^* = RP + \xi \tag{5}$$

as a relation between the coordinates of the same point in body fixed frame ($P \in F$) and in the world frame ($P^* \in F^*$).

Let p is the image of the point P^* and p^* is the image of the same point viewed when the camera is aligned with frame F^* (see fig.2). When all target points lie in a single planar surface one has [1]

$$p_i \cong \left(R^T + \frac{tn^{*T}}{d^*} \right) p_i^*, \quad i = 1, \ldots, k, \tag{6}$$

where $t = -R^T\xi$. n and n^* are the unit vectors normal to respectively the actual and desired image planes. The distance between the object and the desired plane is d^*. The projective mapping $H := \left(R^T + \frac{tn^{*T}}{d^*} \right)$ is called a homography matrix, it relates the images of points on a target plane when viewed from two different poses (defined by the coordinate systems F and F^*).

[1] Most statements in projective geometry involve equality up to a multiplicative constant denoted $\cong$.

More details on the homography matrix could be found in (Malis, 1998). The homography matrix contains the pose information (R, ξ) of the camera whose extraction can be quite complex (Malis, 1998; Zhang and Hanson, 1995; Weng *et al.*, 1992). However, one quantity $r = \frac{d}{d^*}$ (the ratio between actual distance to object d and desired distance d^*) can be calculated easily and directly:

$$r = \det(H) = \det(R^T + \frac{tn^{*T}}{d^*}) = 1 + \frac{n^T t}{d^*}.$$

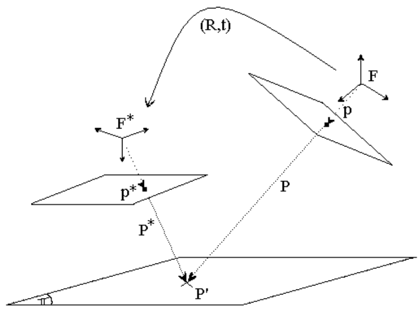

Fig. 2. Camera projection diagram showing the desired (F^*) and the current (F) frames

3.2 Visual Servoing Control Strategy

To simplify the derivation, it is assumed that the camera fixed frame coincides with the body fixed frame F.

Let P' denote the observed point of reference of the planar target, and P^* be the representation of P' in the camera fixed frame at the desired position (Figure 2). The visual servoing problem considered is:

Find a smooth force feedback u depending only on the measurable states (the observed point p,the homography matrix H, the translational and angular velocities (V, Ω), *and the estimated parameters (R,r) from the homography matrix (H) which provide a partial pose estimation), such that the following error*

$$(\epsilon = R(P - R^T P^*), \quad \sigma = \phi - \phi_d)$$

is asymptotically stable.

The angles ϕ and ϕ_d represent respectively the actual and the desired yaw angles. The relationship between the Euler angles and the rotation matrix is[2]:

[2] The following shorthand notation for trigonometric function is used:

$$c_\beta := \cos(\beta), \quad s_\beta := \sin(\beta), \quad t_\beta := \tan(\beta).$$

$$R = \begin{pmatrix} c_\theta c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\psi s_\theta s_\phi - s_\psi c_\phi \\ -s_\theta & s_\psi c_\theta & c_\psi c_\theta \end{pmatrix}. \tag{7}$$

Note that the two error terms (ϵ and σ) are not defined in terms of visual information.

Following (Malis, 1998), the camera can be controlled in the image space and in the Cartesian space at the same time. They propose the use of three independent visual features, such as the image coordinates of the target point associated with the ratio r delivered by determinant of the homography matrix. Consequently, let us consider the reference point P' lying in the reference plan π and define the scaled cartesian coordinates using visual information as follow:

$$P_r = \frac{{n^*}^T p^*}{n^T p} r p$$

Knowing that

$$\frac{||P||}{||P^*||} = \frac{{n^*}^T p^*}{n^T p} r,$$

it follows that we can reformulate the error ϵ in terms of available information, so let us define

$$\epsilon_1 = R\left(\frac{{n^*}^T p^*}{n^T p} r p - R^T p^*\right) \tag{8}$$

From the above discussion and equations describing the system dynamics, the full dynamics of the error ϵ_1 may be rewritten as

$$\dot{\epsilon}_1 = \rho v \tag{9}$$

$$m\dot{v} = -uRe_3 + mge_3 \tag{10}$$

$$\dot{R} = R\mathrm{sk}(\Omega) \tag{11}$$

$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \Gamma \tag{12}$$

where $\rho = \frac{1}{||P^*||} < 1$[3] is an unknown and non identifiable parameter (see Appendix 1 for the identifiability test of the parameter ρ). To avoid the problem of the unknown parameter ρ, we will present a control strategy based on separating the translational dynamics from the rotational dynamics leading to time scale separation between the attitude and linear dynamics.

Define

$$\delta := \epsilon_1 + v \tag{13}$$

Let S_1 be the first storage function for the backstepping procedure. It is chosen for the full linear dynamics Eqn's 9-10

$$S_1 = \frac{1}{2}||\delta||^2 + \frac{1}{2}||v||^2 \tag{14}$$

[3] The distance P^* must be greater than 1 which is the focal length of the camera.

Taking the time derivative of S_1 and substituting for Eq. 9 and 10 yields

$$\frac{d}{dt}S_1 = \rho\delta^T v + (\delta + v)^T(mge_3 - uRe_3) \quad (15)$$

Applying classical backstepping one would assign a virtual vectorial control for $\frac{1}{m}(uRe_3)^d$

$$u(Re_3)^d := mge_3 + mv + m\delta \quad (16)$$

This choice is sufficient to stabilize S_1 if the term $u(Re_3)^d$ were available as a control input. If uRe_3=$u(Re_3)^d$ then

$$\dot{S}_1 = -||\delta||^2 - (2-\rho)\delta^T v - ||v||^2$$

is negative definite $\forall\rho < 1$.

Note that the vectorial input can be split into its magnitude u, and its virtual (or desired) direction R_de_3, that defines two degrees of freedom in the airframe attitude dynamics (Eqn's 11-12):

$$|u| = ||mge_3 + mv + m\delta||$$

and the desired direction

$$R_de_3 = \frac{mge_3 + mv + \delta}{|u|} \quad (17)$$

Note that the full orientation matrix R_d is obtained by solving (ψ, θ, ϕ) using Eq. 7 subject to the constraint given by the specification of ϕ_d. The desired direction vector R_de_3 in equation 17 is not subject to any restrictions. In the next section, the orientation will be limited to small values of Euler angles.

4. LIMITING THE UAV ORIENTATION

In the theoretical developments based on the backstepping (Hamel and Mahony, 2000), the proposed law of control assures an exponential convergence towards the desired position. It seems to us, however, that this type of convergence is not recommended when the vehicle is initially far from the desired position. Indeed, the dynamic model based on quasi-stationary conditions (hovering conditions) is not valid anymore, because the dynamics of such a convergence will provoke a different flight mode. Moreover, the target image may leave the field of view of the camera during the evolution of the vehicle. To avoid such situations, it is necessary to insure that the focal axis of the camera is close to the gravity direction. In the sequel, we propose to use small gains technique (for example the technique of saturation functions presented by Teel in (Teel, 1992)). This technique seems well adapted to our problem. Indeed, if the orientation is saturated, we can insure that the robot will remain in quasi-stationary manoeuvres during all the operation.

The orientation R_de_3 is a function of the terms v and δ. In order to limit the orientation, we add a saturation on the two terms v and δ. Therefore, Eq. 16 becomes

$$uR_de_3 = mge_3 + m\,\mathrm{Sat}_2(v + \mathrm{Sat}_1(\delta)) \quad (18)$$

where $\mathrm{Sat}(x)$ is a continuous, nondecreasing saturation function satisfying:

- $x^T\mathrm{Sat}_i(x) > 0$ for all $x \neq 0$.
- $\mathrm{Sat}_i(x) = x$ when the components of the vector x are smaller than L_i $(|x_{(.)}| \leq L_i$.
- $|\mathrm{Sat}_i(x)| \leq M_i$ for all $x \in \mathbb{R}$.

Proposition 4.1. The following choice of the saturation functions (Teel, 1992)

$$M_i < \frac{1}{2}L_{i+1}; \quad \frac{1-\rho}{2}L_{i+1} \leq L_i \leq M_i$$

ensures global stabilization of the linear dynamics when equation 18 is used as control input of the translational dynamics

Proof Recalling Eq. 10 and Eq. 18, it yields

$$\dot{v} = -\mathrm{Sat}_2(v + \mathrm{Sat}_1(\delta))$$

Consider the storage function $S_v = \frac{1}{2}||v||^2$. The derivative of S_v is given by

$$\dot{S}_v = -v^T\mathrm{Sat}_2(v + \mathrm{Sat}_1(\delta))$$

Using conditions on Sat_i coupled with the fact that $M_1 \leq L_2$, it follows that $\dot{S}_v < 0$ $(\forall|v_{(.)}| \geq \frac{1}{2}L_2)$ ($v_{(.)}$ represents a component of the vector v). Consequently, it exist a finite time T_1 after which all components of the linear velocity vector $v_{(.)} \leq \frac{1}{2}L_2$ $(\forall t \geq T_1)$. The control law Eq. 18 becomes then

$$u_4R_de_3 = mge_3 + m(v + \mathrm{Sat}_1(\delta)), \quad \forall t \geq T_1$$

Now consider the evolution of the term δ for $t \geq T_1$. Let S_δ the storage function associated with the term δ $(S_\delta = \frac{1}{2}||\delta||^2)$. Deriving S_δ it yields

$$\dot{S}_\delta = \delta^T\left((\rho - 1)v - \mathrm{Sat}_1(\delta)\right)$$

Using the second condition of the proposition, one can observe that the components of the vector δ become smaller than M_1 after a finite time T_2. After T_2, the control law becomes

$$uR_de_3 = mge_3 + m(v + \delta), \quad \forall t \geq T_2$$

insuring exponential stability after the time T_2. $\triangle$

Using the saturated control law (Eq.18), the derivative of the first storage function becomes

$$\dot{S}_1 = -||\delta||^2 - (2-\rho)\delta^T v - ||v||^2 \\ - (\delta + v)^T|u|^s(\tilde{R} - I)R_de_3$$

where

$\tilde{R} = RR_d^T$; and $|u|^s = ||mge_3 + m\,\mathrm{Sat}_2(v + \mathrm{Sat}_1(\delta))||$

According to the above proposition, the system with such a saturated input is globally asymptotically stable if the new error term $\tilde{R} - I$ converges to zero. Now, it only remains to control the attitude dynamics involving the error $\tilde{R} - I$.

5. ATTITUDE DYNAMICS CONTROL

The next step of the control design involves the control of the attitude dynamics such that the error $\tilde{R} - I$ converges exponentially to zero. We will use a quaternion representation of the rotation to obtain a smooth control for $\tilde{R}$. The attitude deviation $\tilde{R}$ is parameterized by a rotation $\tilde{\gamma}$ around the unit vector $\tilde{k}$. Using Rodrigues' formula (Murray *et al.*, 1994) one has

$$\tilde{R} = I + \sin(\tilde{\gamma})\mathrm{sk}(\tilde{k}) + (1 - \cos(\tilde{\gamma}))\mathrm{sk}(\tilde{k})^2$$

The quaternion representation describing the deviation $\tilde{R}$ is given by (Murray *et al.*, 1994)

$$\tilde{\eta} := \sin\frac{\tilde{\gamma}}{2}\tilde{k}, \qquad \tilde{\eta}_0 := \cos\frac{\tilde{\gamma}}{2}; \text{ with } ||\tilde{\eta}||^2 + \tilde{\eta}_0^2 = 1$$

The deviation matrix $\tilde{R}$ is then defined as follows

$$\tilde{R} = (\tilde{\eta}_0^2 - ||\tilde{\eta}||^2)I + 2\tilde{\eta}\tilde{\eta}^T + 2\tilde{\eta}_0\mathrm{sk}(\tilde{\eta}) \tag{19}$$

The attitude control objective is achieved when $\tilde{R} = I$. From Eqn 19 this is equivalent to $\eta = 0$ and $\tilde{\eta}_0 = 1$. Indeed, it may be verified that

$$||\tilde{R} - I||_F = \sqrt{\mathrm{tr}((R - I)^T(\tilde{R} - I))} = 2\sqrt{2}||\tilde{\eta}|| \tag{20}$$

Based on this result, the attitude control objective is to drive $\tilde{\eta}$ to zero. Differentiating (η, η_0) yields

$$\dot{\tilde{\eta}} = \frac{1}{2}(\tilde{\eta}_0 I + \mathrm{sk}(\tilde{\eta}))\tilde{\Omega}, \qquad \dot{\tilde{\eta}}_0 = -\frac{1}{2}\tilde{\eta}^T\tilde{\Omega} \tag{21}$$

where $\tilde{\Omega}$ denotes the error angular velocity

$$\tilde{\Omega} = R_d(\Omega - \Omega_d) \tag{22}$$

and Ω_d represents the desired angular velocity. In order to find the desired angular velocity, we have to consider the time derivative of the desired orientation $R_d e_3$

$$\dot{R}_d = R_d\mathrm{sk}(\Omega_d); \quad \dot{R}_d e_3 = R_d e_3 \mathrm{sk}(\Omega_d) \tag{23}$$

Since differentiating the direction Rde_3 involves the use of the unknown parameter ρ which is non identifiable (see Appendix 1), we will design a control law with a high gain virtual control $\tilde{\Omega}^v$. In this way we can neglect the effect of the time derivative of $R_d e_3$.

Then, by choosing the virtual control as

$$\tilde{\Omega}^v \approx \Omega^v = -2k\tilde{\eta}_0\tilde{\eta}$$

with parameter k chosen high enough to neglect Ω_d. Let

$$\nu = \frac{1}{k}\Omega + R_d^T\tilde{\eta}_0\tilde{\eta} \tag{24}$$

After tedious calculations, we obtain the time derivative of ν

$$\dot{\nu} = -\frac{k}{2}\tilde{\eta}_0 R_d^T\tilde{\eta} - \frac{k}{2}\tilde{\eta}_0^2\nu \tag{25}$$

Let us define the Lyapunov function candidate for the attitude deviation :

$$S_2 = \frac{1}{2}||\tilde{\eta}||^2 + \frac{1}{2}||\nu||^2. \tag{26}$$

Taking the time derivative of S_2 and using (25), we obtain

$$\dot{S}_2 = -\frac{k}{2}\tilde{\eta}_0^2||\tilde{\eta}||^2 - \frac{k}{2}\tilde{\eta}_0^2||\tilde{\nu}||^2 \tag{27}$$

This completes the control design for the attitude dynamics, since the time derivative of the storage function in (27) is definite negative. Then the input of the new control law (eq. 18) limiting the orientation ensures the exponential stability of the orientation dynamics and asymptotic stability of the linear dynamics.

6. SIMULATION RESULTS

In order to evaluate the efficacy of the proposed servoing technique with orientation limits, simulation results for a hovering robot are presented. The experiment considers a basic stabilization mission. The target is composed from five points: four on the vertices of a planar square and one on its center. The available signals are the pixel coordinates of the five points observed by the camera.

The parameters used for the dynamic model are $m = 0.6$, $\mathbf{I} = \mathrm{diag}[0.4, 0.4, 0.6]$ and $g = 10$. Initially, the robot is assumed to hover at $(10, 15, -12)$.It is assumed that the plane is perpendicular to the line of sight (*i. e.* the unit vector normal to the target plane is equal to the direction of the gravity $n^* = e_3$)

We will compare the results of the new control law (with orientation limits) versus the evolution of the states in the control law (without orientation limits) developed in (D.Suter *et al.*, 2002). One can notice (Figure 3) that the time of convergence for the states following the new law of control is longer than the previous law, but instead we see that the variation of the Euler angles are restricted to small values (in this case, in the order of 10^{-3}rad).

7. CONCLUSION

In this paper we presented a new control law based on previous work of (D.Suter *et al.*, 2002). Our new contribution is limiting the robot orientation to ensure small values of Euler angles,

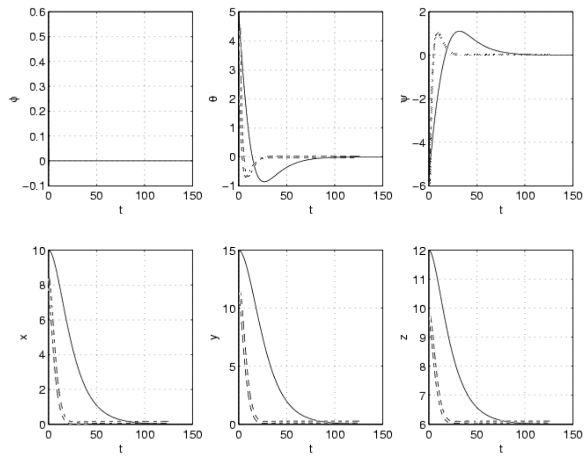

Fig. 3. Evolution of the system states (the 3 Euler Angles [radian] and the 3 coordinates [meter]) in the 2 control laws: without limiting its orientation (Dashed Lines-Angles in 10^{-1}rad), and the new control law (Full Lines-Angles in 10^{-3}rad)

therefore the dynamics of the flying vehicle will be always applicable to the hover conditions (quasi-stationary manoeuvres) and the object will remain at all time in the field of view of the camera.

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APPENDIX 1: TEST FOR THE IDENTIFIABILITY OF ρ

In this appendix, we will apply the conditions given by (Lecourtier and Lamnabhi-Lagarrigue, 1987) to test the identifiability of the parameter ρ. The test is known as the *generating series test*, it is based on power series associated with the system. Rewriting the system Eqn's 9-12 using the following notation $x_1 = \epsilon_1, x_2 = v, x_3 = R$, and $x_4 = \Omega$:

$$\dot{x} = f_0(x, \rho) + f_1(x, \rho)u + f_2(x, \rho)\Gamma \qquad (28)$$

where

$$f_0 = \begin{bmatrix} 0 & \rho x_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & x_3 \mathrm{sk}(x_4) & 0 \\ 0 & 0 & 0 & -\mathbf{I}^{-1}x_4 \times \mathbf{I}x_4 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 0 \\ -\dfrac{1}{m} u x_3 e_3 \\ 0 \\ 0 \end{bmatrix} \qquad f_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{I}^{-1} \end{bmatrix}$$

To simplify the test, we eliminate the term ge_3 from the equation. This elimination wont alter in any way the result of this test. Let us consider the vector fields associated with the last system:

$$F_0(x, \rho)[.] = \rho x_2 \frac{\partial}{\partial x_1} + [x_3 \mathrm{sk}(x_4)] \frac{\partial}{\partial x_3} - [\mathbf{I}^{-1} x_4 \times \mathbf{I} x_4] \frac{\partial}{\partial x_4}$$

$$F_1(x)[.] = \frac{1}{m} x_3 e_3 \frac{\partial}{\partial x_2}$$

$$F_2(x)[.] = \mathbf{I}^{-1} \frac{\partial}{\partial x_4}$$

Then we compute the Lie derivatives of any output function y and evaluate it at $x = x^*$ where $x_1 = x_2 = x_4 = 0$, $x_3 = I$: $F_1(F_0(x^*, \rho)[y])$, $F_2(F_0(x^*, \rho)[y])$, $F_2(F_0(F_0(x^*, \rho)[y]))$. The parameter will be identifiable if the system has a unique solution for ρ. In our case, the Lie derivatives all vanish at $x = x^*$ so the system is *structurally non identifiable*.